A self-adaptive model-order reduction algorithm for nonlinear eddy current problems based on quadratic-linear modeling

Daniel Klis, Stefan Burgard, Ortwin Farle, and Romanus Dyczij-Edlinger

Laboratory for Electromagnetic Theory, Saarland University, Saarbrücken, D-66123
Email: edlinger@lte.uni-saarland.de

Finite-element time-domain simulations of nonlinear eddy current problems require the solution of a large, sparse system of equations at every time step. Model-order reduction is a powerful tool for reducing the computational effort for this task. In this paper, an adaptive order-reduction methodology with error control is proposed. In contrast to previous approaches, it treats the nonlinearity without simplification, by rewriting the original equations as a quadratic-linear system.

***Index Terms*—Finite element analysis, Eddy currents, Reduced order systems, Nonlinear equations, Adaptive algorithms.**

I. Introduction

Electromagnetic devices exhibiting eddy current phenomena have been successfully analyzed by the finite-element (FE) method. However, the resulting systems of equations may comprise several millions of degrees of freedom (DoF), so that their solution is computationally expensive. In the linear case, projection-based model-order reduction (MOR) provides a powerful methodology for reducing memory consumption and computational times. For the solution of nonlinear models, however, the need to assemble the equations on the original, high-dimensional space results in poor speed-up.

To overcome this limitation, state-of-the-art methods, such as the TPWL approach [1] or the DEIM [2], employ affine approximations to the nonlinearity. This class of methods has in common that the affine approximation is based on a training process, which selects a concrete set of parameter values. Despite their usefulness in the trained region, such methods fail when the system state leaves the trained region of state-space and the system behavior changes. A different approach to nonlinear MOR was presented in [3]: The nonlinear equations are reformulated by introducing auxiliary equations in such a way that the resulting system is quadratic-linear (QL) in its variables. For eddy current problems, a non-adaptive MOR strategy based on QL systems was presented in [4]. It computes the projection matrix by an offline training process. The particular treatment of the nonlinearity in [4] does not extend to steady-state calculations.

This contribution presents a QL formulation for nonlinear eddy current problems including the stationary limit. The resulting adaptive method employs a reduced-order model (ROM) to simulate a trajectory in the time-domain, and it verifies the solution at each time-step by means of an error indicator. Only if the error measure exceeds a user-defined threshold, the FE model is solved to enhance the reduced state-space. Thus, the procedure achieves significant speedup compared to conventional FE simulation.

II. The quadratic eddy current model

Following [3], the nonlinear descriptor system

$$\mathbf{E}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \mathbf{f}(\mathbf{x}) + \mathbf{B}\mathbf{u} \tag{1}$$

is reformulated such that it is at most quadratic in the state variables. The resulting system of quadratic-linear differential algebraic equations (QLDAE) is of the form

$$\mathbf{E}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{H}\mathbf{x}\otimes\mathbf{x} + \mathbf{B}\mathbf{u}, \tag{2a}$$

$$\mathbf{y} = \mathbf{C}^T\mathbf{x}, \tag{2b}$$

wherein $\mathbf{E}, \mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are constant matrices, and $\mathbf{H}$ is a third-order tensor. This representation is exact and does not contain any approximation to the nonlinearity. Furthermore, it allows for projection-based MOR, as in the linear case. Using a suitable projection matrix $\mathbf{V}$ so that

$$\mathbf{x} \approx \mathbf{V}\hat{\mathbf{x}}, \tag{3}$$

the ROM reads

$$\hat{\mathbf{E}}\frac{\mathrm{d}}{\mathrm{d}t}\hat{\mathbf{x}} = \hat{\mathbf{A}}\hat{\mathbf{x}} + \hat{\mathbf{H}}\hat{\mathbf{x}}\otimes\hat{\mathbf{x}} + \hat{\mathbf{B}}\mathbf{u}, \tag{4a}$$

$$\mathbf{y} = \hat{\mathbf{C}}^T\hat{\mathbf{x}} \tag{4b}$$

with

$$\hat{\mathbf{X}} = \mathbf{V}^T\mathbf{X}\mathbf{V}, \qquad \mathbf{X} \in \{\mathbf{A}, \mathbf{E}\}, \tag{5a}$$

$$\hat{\mathbf{Y}} = \mathbf{V}^T\mathbf{Y}, \qquad \mathbf{Y} \in \{\mathbf{B}, \mathbf{C}\}, \tag{5b}$$

$$\hat{\mathbf{H}} = \mathbf{V}^T\mathbf{H}\mathbf{V}\otimes\mathbf{V}. \tag{5c}$$

The $\boldsymbol{A}V$-$\boldsymbol{A}$ formulation of the nonlinear eddy current problem [5] leads to the system of partial differential equations

$$\operatorname{curl}\nu\operatorname{curl}\boldsymbol{A} + \frac{\partial}{\partial t}\sigma\left(\boldsymbol{A} + \operatorname{grad}V\right) = 0, \qquad \sigma > 0, \tag{6a}$$

$$\operatorname{div}\left(\sigma\boldsymbol{A} + \sigma\operatorname{grad}V\right) = 0, \qquad \sigma > 0, \tag{6b}$$

$$\operatorname{curl}\nu\operatorname{curl}\boldsymbol{A} = \boldsymbol{J}_i, \qquad \sigma = 0,, \tag{6c}$$

in the computational domain $\Omega \subset \mathbb{R}^3$. Therein, $\boldsymbol{A}$ denotes the magnetic vector potential, V the electric scalar potential, $\boldsymbol{J}_i$ the imprinted electric current density, and σ the electric conductivity. Voltages are computed after [6]. The magnetic reluctivity $\nu(\cdot)$ models saturation effects via

$$\boldsymbol{H} = \nu(\|\boldsymbol{B}\|)\boldsymbol{B}, \tag{7}$$

where $\boldsymbol{H}$ is the magnetic field strength and $\boldsymbol{B}$ the magnetic flux density. To derive a formulation that fits into the framework (2), we choose $\boldsymbol{A}$, V, and ν as unknowns. FE discretization of (6) leads to a finite-dimensional system of equations which, however, does not determine ν. To complete the QLDAE model, (7) must be included in suitable form.

The reluctivity function ν is usually determined from measurement data. The mathematical model, in contrast, requires a closed-form representation. Thus, we approximate ν by a weighted superposition of cubic polynomials:

$$\nu = \sum\nolimits_i s_i \left(c_{3i}B^3 + c_{2i}B^2 + c_{1i}B + c_{0i}\right), \tag{8}$$

$$B = \|\operatorname{curl}\boldsymbol{A}\|, \tag{9}$$

with c_{ki} obtained from spline interpolation of the measurement data. The weights s_i are based on the algebraic sigmoid function

$$\varsigma(x) = Kx/\sqrt{1+(Kx)^2} \tag{10}$$

with steepness $K \in \mathbb{R}$. The entries of the FE matrix from (6) are integrated numerically. Hence (8) and (9) provide additional equations for each integration point lying in the nonlinear material domain. While (8) fits into the QLDAE framework after minor modifications, the norm in (9) does not. However, given the FE expansion of $\boldsymbol{A}$,

$$\boldsymbol{A} = \sum\nolimits_j \boldsymbol{w}_j a_j \tag{11}$$

with trial functions $\boldsymbol{w}_j$ and coefficients a_j, the square of (9),

$$B^2 = (\sum\nolimits_j \operatorname{curl}\boldsymbol{w}_j a_j)^2, \tag{12}$$

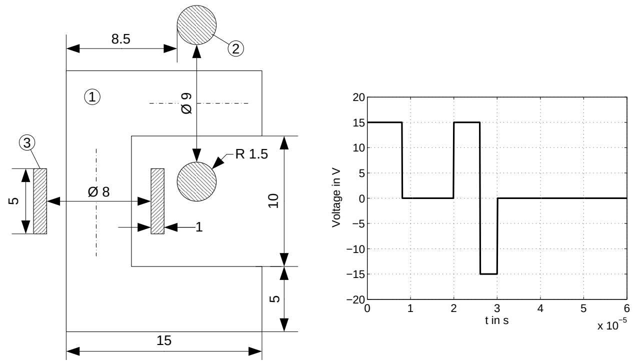
is a polynomial at most quadratic in its unknowns and hence a suitable representation.

The adaptive algorithm constructs the ROM successively based on Proper Orthogonal Decomposition. At each time-step, the solution of the ROM and a residual-based error indicator are computed. If the indicator value is above a certain threshold, the original model is solved to enhance the projection basis. Details will be given in the full paper.

III. Numerical example

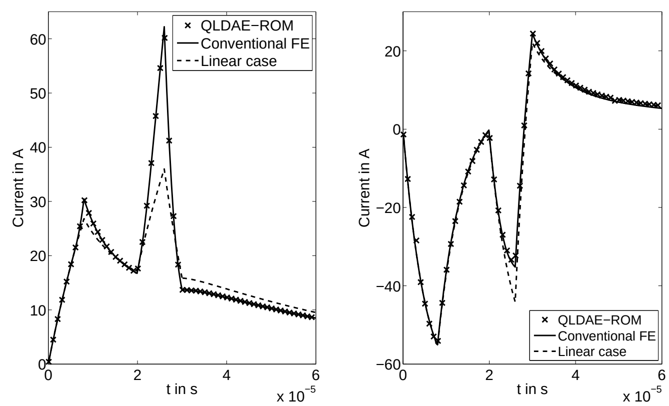
We consider the transformer depicted in Fig. 1(a). The voltage signal of Fig. 1(b) is applied to the primary coil, while the secondary coil is short-circuited. The coil currents are simulated for 60 μs using 600 equally spaced time-steps. In Fig. 2, good agreement between the ROM solutions and FE results is observed.

The conventional FE system features 92443 DoF. The QLDAE system is obtained by adding 19082 unknowns corresponding to 14 constitutive equations per integration point (6 summands in (8)). Note that the non-reduced QLDAE system is never solved; it is solely used for the projection (4). The resulting ROM features 176 DoFs. Total simulation time for a MATLAB code on an Intel i5-4670K processor is 662 s, including 33 evaluations of the conventional FE model in the adaptive process. Standard FE simulation takes 2856 s. The speed-up factor of 4.3 is promising but less impressive than in the linear case. This is partly because of the prototype state of the implementation and partly because of the presence of the third-order tensor.



(a) Geometry. (b) Excitation voltage.

Fig. 1. Two coils on nonlinear ferrite core (1) of quadratic cross-section. Dimensions are in mm. (2): secondary coil of relative magnetic permeability $\mu_r = 1000$ and electrical conductivity $\sigma = 10^7$ S/m. (3): primary coil of $\mu_r = 1$ and $\sigma = 0$ S/m.



(a) Primary coil. (b) Secondary coil.

Fig. 2. Coil currents. Comparison between the QLDAE-ROM, conventional FE simulation, and the linear case.

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